Remarks to the author by Harold Davenport

(Contained in letters dated 23rd June 1966 and 21st February 1967)

1. A test case for the possibility of getting more than in the classical
Waring’s problem is the following: What can we say about the number
of solutions in polynomials of

\[ x_1^3 + x_2^3 + x_3^3 = y_1^3 + y_2^3 + y_3^3 \]

with \(|x_i| \leq p^\nu, |y_i| \leq p^\nu|\)?

Can we get \(O(p^{3\nu+\epsilon})\)?

In the classical case we can only get what corresponds to \(O(p^{2\nu})\) roughly
and this is a barrier to further progress.

Hua’s theorem for \(k = 3\) would correspond to the fact that the number
of solutions of

\[ x_1^3 + \cdots + x_4^3 = y_1^3 + \cdots + y_4^3 \]

is about \(p^{5\nu}\).

2. The condition \(p > k\) raises tantalizing problems. Kubota consulted me
as to what happens when \(k = p + 1\) for example.

It may be that the asymptotic formula is not true in its normal form
otherwise. My impression is that if \(p = k\), the exponential sum is either
0 or large on each minor arc. But I was unable to find a satisfactory
way of attacking this case.