Remarks to the author by Harold Davenport

(Contained in letters dated 23rd June 1966 and 21st February 1967)

1. A test case for the possibility of getting more than in the classical Waring's problem is the following: What can we say about the number of solutions in polynomials of

$$x_1^3 + x_2^3 + x_3^3 = y_1^3 + y_2^3 + y_3^3$$

with $|x_i| \leq p^\nu, |y_i| \leq p^\nu$?

Can we get $\mathrm{O}(p^{3\nu+\epsilon})$?

In the classical case we can only get what corresponds to $\mathrm{O}(p^{\frac{7}{2}\nu})$ roughly and this is a barrier to further progress.

Hua's theorem for $k = 3$ would correspond to the fact that the number of solutions of

$$x_1^3 + \cdots + x_4^3 = y_1^3 + \cdots + y_4^3$$

is about $p^{5\nu}$.

2. The condition $p > k$ raises tantalizing problems. Kubota consulted me as to what happens when $k = p + 1$ for example.

It may be that the asymptotic formula is not true in its normal form otherwise. My impression is that if $p = k$, the exponential sum is either 0 or large on each minor arc. But I was unable to find a satisfctory way of attacking this case.