From the Editor

This issue of MAA FOCUS has been a challenge to finish. We were about to wrap up the issue when two things happened: Richard Guy passed away at the prime age of 103 and the coronavirus took a firm hold in the United States. Things have changed very quickly over the past couple of weeks. A conference at which I was scheduled to present was cancelled two days before I was to leave, sections began cancelling their spring meetings, and campuses closed.

Lamar University followed many other universities and colleges, cancelling face-to-face classes through May. We are still discussing what will happen with our summer courses, but many of us are thinking about how to transition summer classes fully online.

I’ve been watching MAA members come together in such lovely ways—posts on MAA Connect asking and offering support, and blog-like posts about how courses are evolving. Friends meeting in Google Hangouts to work on research (or have a cocktail hour). Posts asking for support or commiseration as we all struggle with these challenges.

Institutions of higher education and K–12 education are being asked to do something that we’ve never been asked to do before—with as little as two days notice, figure out how to transition all learning into an online environment for students who might not have internet access beyond their phone. We are trying to use best practices, but are also just trying to get through. And it’s hard. Really hard. I know so many teachers, faculty and staff who are coping with a partner no longer working, kids home from school, worries about their health or the health of their family and friends. And I know the support I am finding online is helping me stay connected in this era of social distancing (or shelter-in-place). While I watch the numbers rise, I continue to hope that this pandemic passes quickly and with a minimal loss of life. I hope you stay as healthy as possible, and that we can share stories of success soon. Be well, friends and readers.
Richard K. Guy (1916–2020)

A much beloved member of the MAA community, Richard Guy, passed away on March 9, 2020. Richard was an MAA lifetime-member and was a religious attendee at national meetings, a frequent contributor to our journals, author of two MAA books, and served on numerous committees during his 52 years in the Association.

Richard Kenneth Guy was born September 30, 1916 in Warwickshire, England and was the only child of two schoolteachers. After education at Cambridge (Caius College) and Birmingham University he began teaching at a British grammar school. Looking for deeper engagement with mathematics he embarked on a university career that took him from London University to the University of Malaya, Singapore, to the Indian Institute of Technology in New Delhi and, finally, in 1965, to the University of Calgary where he spent the rest of his life.

Richard's mathematical interests were broad. He contributed to number theory, geometry, combinatorics, combinatorial game theory, graph theory and recreational mathematics. We will perhaps best remember him through his many books. His fifteen-year collaboration with Elwyn Berlekamp and John Conway resulted in *Winning Ways for your Mathematical Plays* which introduced the mathematics community to the wonders of combinatorial games and Sprague-Grundy theory (which Richard had independently rediscovered). He was an inveterate collector of interesting problems and he had a terrific ability to frame them as intriguing, appealing mysteries—his *Unsolved Problems in Number Theory* is a beguiling invitation to that discipline. Of late he had turned his attention to triangle geometry and to finishing his Carus monograph (co-authored with Bud Brown) *The Unity of Combinatorics* due out in May.

Richard discovered mountaineering during his time in India on an excursion to the Himalayas. His interest in hiking, climbing, and environmentalism flourished in the Canadian Rockies after he moved to Calgary. The Alpine Club of Canada maintains the *Louise and Richard Guy Hut* on Mont des Poilus in British Columbia in honor of Richard and his late wife Louise. Louise shared Richard's love of the outdoors and, over the seventy years of their marriage, shared their home and hospitality with a stream of mathematical visitors. He possessed a longstanding interest in chess endgames; for a time he edited a column in a British chess magazine and he maintained an encyclopedic archive of problems. He had a sense of humor composed in equal parts of wordplay, mathematical cleverness, and self-deprecation; read any
of his books or articles and you’ll experience it. In person he would flash a mischievously proud grin when he pulled one of these off. In his Lester Ford Award-winning *Monthly* article, “The Strong Law of Small Numbers,” he exhibited a couple score examples; some were true patterns and the rest were coincidences. He wittily explained to us that there just are not enough small numbers to handle all the demands we place upon them!

Richard officially retired from the University of Calgary in 1982 but he maintained an office there and went in to work nearly every day until the end of his life. He would say, “I didn’t actually retire, they just stopped paying me.” But he never did it for the pay: he would say—in spite of his 300 papers and 12 books—that he was not a mathematician, but an amateur. He was well aware that the Latin root of that word means “love” and he would tell you that he was a lucky man getting to do what he loved for 103 years. We who loved him were fortunate to have him for that long.

*MAA FOCUS* asked some of Richard’s friends and admirers for personal reminiscences.

**Ezra (Bud) Brown**  
**coauthor of* The Unity of Combinatorics* with Richard Guy**

A referee’s report on a certain paper of mine said that the paper was fine as is “… but here are a few suggestions that might improve the paper.” The report concluded, “It’s OK if you tell the author my name.” The suggestions improved the paper to the point where it was selected for an MAA writing award. That referee was Richard Guy, and he was in the audience at MathFest 2003 when the paper was honored. This was the beginning of my friendship with this giant of the world of numbers and sequences and patterns and games. Over the next two decades, we met at meetings and went to talks together—always on the front row. He signed my copy of his book *Winning Ways—Book One* with these words: “To Bud Brown, a cheerful mathematician.” Being invited to join Richard as a co-author of *The Unity of Combinatorics* was a great honor that turned into a seven-year adventure. Richard Guy died on Monday, March 9, 2020. He was 103 years young. He lived long enough to see our book finished and sent to the press.

He was not a big man, but he really filled a room. And he, too, was a cheerful mathematician.

**Don Albers**  
**former MAA Director of Publications and indefatigable acquisitions editor, on Richard Guy the intrepid hiker**

I had only been MAA Director of Publications for a few months when Richard called and told me that I had to come to Calgary for a week and help him and John Selfridge finish *The Lighter Side of Mathematics*, a Spectrum book. I agreed to come help, although the idea of helping Richard, an accomplished writer, would strike many as hilarious. I added a condition—once the manuscript review was completed, we would go hiking on some of the trails around Lake Louise. With one day to go, we finished. Off we went to Banff and Lake Louise—a spectacular lake embedded in beautiful mountains. After a few hours of hiking, the trail ascended sharply to the west, and our pace slowed. Richard said “we’re almost there.” We soon arrived at the Bee Hive, which turned
Richard speaking at MAA MAthFest 2016.


out to be something of a hikers’ cafe. We relaxed over coffee and freshly baked bran muffins. I looked down upon where we had been, and felt pleased with myself.

I thought we would soon be returning to the car. Richard then informed me that we were just getting started going up! I reminded myself that Richard was 76 and I was 25 years younger than that. Certainly I could do as well as him. The trail continued upward to the southeast. Richard moved with ease, which spurred me on, even though my legs were beginning to tire. Our objective was a hikers’ hut in a snowfield, which was more than a thousand feet higher. We continued, but dusk was approaching. Richard found a faster ridge trail that would enable us to get back to the car before dark. He took off like a shot, leaving me in his dust. An hour later I dragged into the parking area, where I found Richard napping in the car. He woke up and told me that I was driving us back to his home in Calgary. So much for my nap! An hour later, we arrived at his home. I slumped into an armchair and struggled to keep my eyes open.

Don Albers on How Author Teams Are Made

Richard Guy had talked about writing the *Unity of Combinatorics* (TUoC) for years, and I was eager to publish it. He was not getting any younger, so my concern about TUoC seeing the light of print was growing with each passing year. I knew that Richard liked Bud Brown’s writing on number theory and combinatorics, and that Richard greatly enjoyed Bud’s musical performances at banquets and his sense of humor. I also was aware that Bud wrote like a ball of fire. The three of us happened to be standing together in one of the halls at the Hartford MathFest in 2013 when the clouds parted, and the solution appeared like a lightning flash. The answer to my problem was to appoint Bud and Richard co-authors of TUoC, so I did on the spot. They shook hands, and the deal was done. The rest is history. I have had many joys in finding authors for MAA books; assembling the Brown-Guy team gave me a real glow.

Rob Curtis
algebraist and combinatorialist,
University of Birmingham (UK)

Richard was a frequent visitor to Cambridge University during my time as a PhD student and then as postdoctoral fellow. He came to work with my then supervisor, John Conway, mainly on mathematical games. That work eventually resulted in the remarkable book *Winning Ways* which was co-authored by the two of them together with Elwyn Berlekamp, who died last year. I well remember their discussions of the *mex*—minimal excludant—of a set of nonnegative integers, leading to their proof that the Grundy numbers for rows of skittles in *Kayles* eventually became periodic with period 12, but only when the length of row exceeded 72. When Conway turned 60 it fell to me to design his birthday cake and I wanted to include a wide range of his mathematical interests. Naturally I included several mathematical games. Standing looking at the cake with Richard I asked him if he’d noticed who were the two players in the ‘dots and boxes’ game illustrated—I had labelled them B and G—he immediately replied “Berlekamp and Guy, of course!”

John had been a close friend of and early collaborator with Richard’s son Mike and, I think it’s fair to say, Richard was quite a father figure to John. Certainly people like me who were closely involved with John benefitted from Richard’s visits as the latter would organize the former and make sure
that he carried out all the necessary chores which John had inevitably left undone.

From a personal point of view, Richard was a significant presence at the birth of the Miracle Octad Generator. I had been trying for some time to produce a diagram of the Steiner system $S(5,8,24)$ and finally realized whilst enjoying a pint of ale in the Cricketers’ Arms that I could reduce the whole thing to just 35 pictures. The next morning I took the resulting diagram to John and together we set about producing a fair copy. We worked late into the night in the math department but then repaired to the guest room in Caius College where Richard and Louise were staying. Midst much wine-bibing and a general party atmosphere the job was completed in the early hours of the morning. When printed the next day and shrunk to a quarter the size of the original, any blemishes due to our shaky hands and general inebriation mysteriously disappeared.

I have the fondest memories of Richard in those days, and was always delighted to bump into him at conferences in more recent years. He was a wonderful asset to our world of mathematics and he will be sadly missed.

Deanna Haunsperger
former MAA President, Carleton College

I don’t actually recall meeting Richard for the first time. But I do remember bumping into him one day in the Minneapolis airport, his flight from Calgary connected there. I was very young, this was, perhaps, 25 years ago. I distinctly remember being thrilled that the famous Richard Guy knew my name. We had a lovely conversation and I was amazed at his modesty; he was Richard Guy and I was nobody. Somehow after that we had a connection; we would find each other at every MathFest and JMM. Sometimes he’d have a puzzle or problem to share, always he’d ask after my family. It became an essential part of national meetings for me to find Richard and check in. Meetings became a little less fun when he stopped attending a couple of years ago. I have always loved his writing voice, it is witty and smart and friendly, but it is nothing to his physical presence. He was kind and funny and caring. Yes, he was a great mathematician, but, even more, he was a great person. I miss him.

Mark Krusemeyer
Carleton College, coauthor of “Partitions with parts occurring at most thrice” with Alex Fink and Richard Guy

One of my enduring memories of Richard will surely be his greeting when I would knock on his door and walk into his office. Whether it had been just a few days or close to a year since I had seen him, he would look up with a gleam in his eye and declare, “I remember you!” I will also remember one of his long-time colleagues assuring me when I mentioned that apparently I was somewhat deficient in mathematical stamina: “Don’t worry, none of us can keep up with Richard!”

Loren C. Larson,
St. Olaf College, coauthor of The Inquisitive Problem Solver with Paul Vaderlind and Richard Guy

Seems like I’ve known Richard Guy all my life. He was part of a larger community of friends in those days who wrote about recreational mathematics—giants such as Martin Gardner, Paul Erdős, Ron Graham, John Conway, Elwyn Berlekamp, and Roger Penrose. Their writing was inspired by their energy and it was contagious. Their articles usually began with a simple theme, an irresistible problem to get you hooked. Then they followed with variations, extensions and conjectures, based on the key ideas. Reading their work was fun and exciting and made me want to be a mathematician, to experience all parts of their joy—the questioning, experimenting, conjecturing, discovering, analyzing, verifying, and sharing.

Years later, when Richard was in his eighties, I had the privilege of working with him on expanding and revising a book of mathematical brain-teasers composed by Paul Vaderlind at the University of Stockholm. Richard recognized their high quality and our aim was to highlight the mathematics behind the solutions. We corresponded by email off and on for three years and on a few occasions I spent time with him in Calgary. I think Richard viewed each of these problems as small-time adventures in mathematics, and he insisted they each be named—the 13-coin problem, the knight-exchange problem, the cookie monster, and so forth. In that way, the problem and the ideas behind it could be remembered and appreciated. Sometimes, for problems that required case-by-case consideration, he would name each case as a problem in
including the Alpine Club of Canada. There is now a Louise and Richard Guy Hut near the base of Mont des Poilus. When he turned 100 his friends and colleagues in Calgary had him whisked to the top of one of his favorite mountains in a helicopter. A classy gesture for a classy Guy!

I’ll always smile when I think of his “Peace is a disarming concept” button. And I’ll miss him, as will so many of us who were fortunate to spend a little time now and then in his orbit.

Richard Nowakowski
seven-time coauthor and PhD student, Dalhousie University

My first memories of Richard are as an undergraduate taking a number theory class from him. There were six honors students and about twenty regular majors. One day he came in and gave an example of continued fractions, the example being pi. He immediately wrote out the first 30 digits of pi and then proceeded to do the calculations on the board! In all the courses I took from Richard, he was famous for giving ten questions on each assignment, of increasing difficulty. The honors students would meet at 10 pm on Tuesday night for pizza and compare notes. We learnt very quickly that there was one question that was tricky. Sometimes Richard didn’t know the solution himself! Often we would work through the night to get something, anything, on that question. The class was at 8 AM! Which brings me to one of the reasons I became a mathematician. One assignment, I mistook (misread) an easy question for the hard question and saw a really neat approach, which ‘clearly’ had to be the approach Richard meant us to take. I was crestfallen when my friends showed me how easy the actual question was. The next Monday, Richard handed back the assignments and told me that over the weekend we had written a paper! My first paper still has the longest Math Review of all my papers.

Richard was always willing to take the time to explore mathematics with students and I have seen many others whose abilities were honed by Richard’s sharing of his time, expertise, and sense of fun.

Hugh Williams
number theorist, Universities of Manitoba and Calgary

I first met Richard almost 50 years ago at the inaugural meeting of the Manitoba Conference on Numerical Mathematics in 1971 at the University of Manitoba. Richard and I had many interests in common, but as we were in different institutions we did not see each other regularly. In 2001, I moved to the University of Calgary, where of course I saw him much more frequently, and we often chatted about various mathematical matters. I also saw him socially on many occasions. It wasn’t until about 2008 that he brought me a problem that he thought might interest me, and he was certainly right. This began a very fruitful collaboration, which resulted in several
papers on various aspects of linear divisibility sequences and led to our being able to find a natural extension of the Lucas sequences.

Richard had a whimsical and often self-deprecatory sense of humour. He was fond of saying, when asked about his longevity, that one should choose one’s parents wisely. When in 1991 the University of Calgary awarded him an Honorary LL.D., his response was “The University got a bit embarrassed, I think.” [The joke here is that the University, on whose faculty Richard served, would be embarrassed because he did not hold a PhD.] Of course, as he very well knew, the University was immensely proud of him (and still is). Although he had more than 300 publications in such areas as combinatorics, graph theory, geometry, game theory and number theory, he would always say that he was a “Jack of all trades, master of none.” In speaking about his work, he said this:

“I suppose my small claim to fame arises from the fact that mathematics owes more to those that ask questions than to those who answer them. Not that I’ve asked many questions, but I’ve collected them from others, especially Paul Erdős, and then hawked them around.”

Mike Bennett and Ben Green have both told me that their early inspiration came from Richard’s book *Unsolved Problems in Number Theory*. This book is a marvelous compilation of problems and commentary that is instantly infectious. It is no exaggeration to say that this remarkable book has stimulated generations of aspiring number theorists.

In the course of his investigations of various sequences, Richard discovered what he wittily referred to as “The Strong Law of Small Numbers.” In his very engaging and influential paper of the same title he discusses 35 examples of patterns that seem to appear when we check small values of $n$. Some work, but many don’t. He concludes there aren’t enough small numbers to meet the many demands made of them.

Through his unflagging compilation of mathematical questions and commentary, Richard has certainly benefitted mathematics immeasurably. He has acted tirelessly as a popularizer of mathematics, not just to enthusiasts and undergraduates, for whom he always had time, but also, through his evident delight in his subject and promotion of problems, to professional mathematicians.

It seems fitting to conclude this compilation of memories with what Richard thought of his life: “I count myself as the luckiest person in the world. I was married to the best wife in the world for 70 years and I was paid for doing what I like doing.”
The news of Richard Guy’s passing was a blow. Not only because he was a dear friend, but also because I knew that the appearance of his last book, *The Unity of Combinatorics*, was imminent and that he would never see it. When I first met Richard decades ago I was too much in awe of him to actually talk, we had a nod-and-smile relationship for a long time. That changed about 15 years ago. I was sitting at an airport gate leaving JMM to come home and Richard in his familiar brown tweed jacket with his ever-present Peace is a Disarming Concept lapel button sat down next to me and asked about the math on the pad of paper in my lap. At the time I had just discovered Geometer’s Sketchpad and was using its capability to combine Euclidean geometry and motion to generate undergraduate research problems, questions like: What’s the locus of centroids of all the triangles that share a circumcircle? With Geometer’s Sketchpad you could make a little movie and observe that locus being generated in real time. It was thrilling to watch.

I don’t remember exactly what problem I was struggling with at that airport gate but it was something close to the above and Richard listened thoughtfully and we spent an hour swapping ideas and pictures. It was clear that he knew about a thousand times as much about geometry as I did and also clear that his brain worked at about twice the speed mine did. But my awe melted away in the face of his kindness and modesty. He was genuinely interested in my ideas and in working together on the problem. He also had a razor-sharp wit and after one of his jokes would flash his disarming, but devilish, grin. It was great fun to do mathematics with him. Eventually he started telling me about the lighthouse problem [2]: What is the locus of the point of intersection of two rotating lighthouse beams? The cited paper is a great place to go to understand Richard’s approach to mathematics and to experience his sense of humor. For another quick taste of the latter, check out the MAA Review of *The Inquisitive Problem Solver* by Richard’s alter ego, Dick Fellow.

When I got home I had an e-mail waiting from Richard with some more ideas about my problem. We continued that e-mail correspondence for a while. He always did me the kindness of pretending that I was knowledgeable about geometry; I think it was enough for him that I clearly loved it. A few years later I was in Calgary visiting Richard to talk about a possible book on combinatorial games. I spent a week with him, every morning we’d go to his office at the University of Calgary. He taught me about Sprague-Grundy theory and we analyzed dozens of games together. Every evening we’d go back to his home and eat one of the dreadful frozen pot pies he favored for dinner, then get back to work. For a time I thought I could understand three-car Dodgerydoo, Richard did me the courtesy of taking seriously the possibility that I did. (Of course, I didn’t. I think he probably suspected as much all along but was too polite to say so.) We never got the book put together. In spite of that, it was one of the best mathematical weeks in my life.

*The Unity of Combinatorics* is the latest volume in the MAA Carus series and its genesis was a paper by that name that Richard published in 1995. Richard was reacting to the perception that combinatorics was nothing more than a bag of disconnected clever tricks for toy problems. It is clear today that combinatorics is a mature mathematical discipline with deep problems, subtle results, and intriguing connections to other areas of mathematics. Twenty-five years ago that was
not clear and combinatorics’s connection to recreational mathematics made it seem slightly disreputable and frivolous. This book was first imagined by Don Albers who encouraged Richard to expand his article and recruited Bud Brown as a co-author. The result reflects both authors’ personalities, their mathematical interests and their beguiling expository skills. It’s a pure pleasure to read; the perfect mixture of Richard’s gentle wit, Bud’s down-home, welcoming enthusiasm, and both authors’ deep knowledge of, and absolute joy in, the combinatorial landscape.

Let me give you a taste. Suppose you want to find a collection of five-element subsets of the eleven-element set \(\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, X\}\) with the property that each pair of elements occurs together exactly twice. It’s not obvious, at least to me, that such a collection is even possible. A quick count—each of 55 pairs occurring twice is 110 pairs, a five-element set contains ten pairs—will tell you that any such collection will contain 11 sets. But that’s no help in finding it, or even proving it’s possible. It just reassures you that it is not obviously impossible. The Brown-Guy example is given in Table 1, but you’re encouraged to try and construct your own example before peeking.

It is also not obvious why you want to do this. The respectable answer is that it is an example of an (11, 5, 2) symmetric block design, objects that arose in the design of statistical experiments in agriculture. (The parameters correspond to the bolded numbers in the previous paragraph.) The frivolous answer points to the obvious analogy with Kirkman’s Schoolgirl Problem. You are of course wondering which values of \((v, k, \lambda)\) actually correspond to achievable symmetric designs. You should read Chapter 7. I’m more interested in following up on (11, 5, 2) right now. Brown and Guy call this gadget a biplane. It is worthwhile to understand why.

Suppose that instead of requiring each pair of elements to occur twice we will be satisfied with a single appearance. As noted above there are 55 pairs and each five-tuple contains ten, so (11, 5, 1) fails the obvious divisibility test and no such object exists. But, to take one example, (91, 10, 1) does not fail and, so, is not obviously impossible. (Those numbers are a clue to what’s happening, but you might not recognize that.) If we think of the elements as points, we are looking for ten-point subsets such that each pair of points is in exactly one subset. Replace “subset” by “line” and you recognize the description of a finite projective plane of order nine. Thus, the (11, 5, 2) biplane. More saliently, perhaps you begin to see Brown-Guy’s “Unity.”

Brown in [1] asked himself how he might draw a useful picture of the (11, 5, 2) biplane. He wanted the picture to reflect some of the symmetries of the design. For example, note that the product of the two five-cycles \((1, 3, 9, 5, 4)(2, 6, 7, 8, X)\) is a permutation of our original set of order five. Note that it preserves the block structure, e.g., 2456X goes to 61478. This corresponds to the five-fold rotational symmetry in the figure. In fact, as Brown and Guy show, the symmetry group of the biplane actually has order 660 and can be shown to be PSL(2,11). Many of these symmetries can be seen directly in Figure 1.

One final unification observation. It’s interesting to notice that \(2^{11} = 1 + 2^2 + 2^3 + (2^3)^2\) and \(3^5 = 1 + 2^2 + 2^3\). It is known that this equality is exactly what is required for the existence of a perfect three-error-correcting binary code. The alphabet has 23 symbols and the codewords are length 12. Similarly, the fact that \(1 + 2^2 + 2^3 - (2^3)^2 = 3^5\) means that there exists a perfect two-error-correcting ternary code. In this case the alphabet has 11 letters and the codewords have length six. Each of these codes can be realized as the row space of a particular matrix. Suppose one were to construct the \(11 \times 11\) incidence matrix for the (11, 5, 2) biplane by putting a 1 in the \((i,j)\) entry if element \(i\) is in subset \(j\) of the biplane and a 0 if not. (NB: The subsets are indexed by their first listed element in Table 1.) This incidence matrix lives inside the code matrix as a submatrix in the case of each of those codes. You are invited to explore why.

All of the above was taken from just one chapter of Brown-Guy, and we have already run into statistics, group theory, linear algebra, coding theory, recreational mathematics, and projective geometry. Perhaps The Ubiquity of Combinatorics would have been a better title. Whatever we call it, it is full of wonders and breathes with Richard’s spirit. It is a fitting memorial to a mathematical giant whom we were lucky to have for 103 years.

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**Table 1**

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Footnotes:


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*Stephen Kennedy (Carleton College) manages acquisitions for MAA Press. Contact him if you’re interested in writing a book for MAA Press: kennedy@maa.org.*