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Vyacheslav Alekseevich Oleinikov (obituary)

On the night of 4–5 March 1989 Vyacheslav Alekseevich Oleinikov, professor of the theory of functions and functional analysis in the faculty of mechanics and mathematics of Moscow State University, died tragically in a fire at his house outside Moscow.

Oleinikov was born on 10 March 1939 in Moscow. In 1956 he enrolled in the faculty of mechanics and mathematics of Moscow State University. After graduating (in 1961) he worked for two years in the Computer Laboratory at Moscow State University (and later on he kept up his interest in applied problems: he published several papers on work connected with economic treaties). In 1963 he became a research student in the faculty of mechanics and mathematics of Moscow State University in the department of number theory (of which A.O. Gel'fond was then the director) and his supervisor was A.B. Shidlovskii.

Oleinikov's main research interests were problems of transcendental numbers, which are closely linked to algebraic and classical transcendental functions and differential equations. In 1929 C.L. Siegel propounded a method (based on the ideas of Hermite and Lindemann) which made it possible to study the arithmetical nature of the values at algebraic points of quite a wide class of entire analytic functions (Siegel's E-functions), which satisfy linear differential equations with polynomial coefficients. In 1955 Shidlovskii developed Siegel's ideas and completely reduced the arithmetical problem of the transcendental nature and algebraic independence of the values of such functions at algebraic points to the function-theoretic independence of these same functions (and their derivatives) over the field of rational functions. However in real situations the application of Shidlovskii's theorem ran into significant difficulties; in particular, it was not possible to investigate fully the question of the independence of $y(\alpha)$, $y'(\alpha)$, $y''(\alpha)$ for the classical E-function

$$y(z)=K_{\lambda,\mu}(z)=\sum_{n=0}^{\infty}\frac{1}{n!\,(\lambda+1)\dots(\lambda+n)(\mu+1)\dots(\mu+n)}\left(\frac{z}{3}\right)^{3n},$$

satisfying the third-order equation

$$y'''+\frac{3(\lambda+\mu+1)}{z}y''+\frac{(3\lambda+1)(3\mu+1)}{z^2}y'-y=0$$

with rationals $\lambda, \mu \neq -1, -2, \ldots$. Oleinikov considered the general concept of the differential reducibility of an algebraic differential equation and gave criteria for reducibility, reducing the question to one of the solution in algebraic functions of certain partial differential equations, explicitly generated by the initial equation. In the end, by applying this to the third-order equation he was able to prove that for algebraic $\alpha \neq 0$ the numbers $y(\alpha)$, $y'(\alpha)$, $y''(\alpha)$ are algebraically dependent if and only if $\lambda \mp 1/3$ and $\mu \pm 1/3$ are simultaneously integers. This approach proved to be applicable to some other classical hypergeometric functions.

We must mention that Oleinikov's research on differential irreducibility, which he continued to the end of his life (regrettably his last results were only published in summary form), is clearly of interest in the context of number theory.

Oleinikov's other research was directly associated with the study of algebraic functions of several variables. He introduced the concept of an algebraic function with separated singularities and he gave explicit representations of such functions. Results on rational algebraic superpositions are close to this work.

Oleinikov was quite sceptical about fashion in mathematics; he knew and loved the classical mathematics of the 19th century (his main papers usually begin with a reference to Hermite or Frobenius). That may explain why his mathematical contacts were rather narrow.

From the time he received his degree of Doctor of Philosophy until the end of his life Oleinikov worked in the department of the theory of functions and functional analysis in the faculty of mechanics and mathematics of Moscow State University. Although he took part in almost all forms of teaching—giving lecture courses (including very original courses on the theory of special functions), supervising research students and undergraduates, holding special seminars, he enjoyed above all else giving lectures to second-year students on the theory of analytic functions. Here he succeeded in presenting some original ideas, which required accurate thinking through and precise execution. For example, all the basic material for the first stage was covered in the first semester, and in the second one he introduced more general questions; this approach, which of course requires extra effort, usually meant that the interesting and exciting sections had to be considered hurriedly at the end of the academic year. Over his 25 years of teaching many hundreds of graduates from the faculty of mechanics and mathematics remember him with gratitude, even though he was strict and demanding. He worked with his pupils, he loved elementary mathematics, and he wrote interesting articles for "Kvant".

Oleinikov was an unusual man; he had a wide range of interests, he was a profound scholar and a connoisseur of works on art, history, and philosophy. He loved and understood music, he played the piano quite well even though he had never learnt it—he simply had an instrument and some scores; he preferred Chopin, Liszt, and Beethoven. In one of the summer months he used to go with his family and friends on a canoeing expedition. He did not like hunting, he was not an enthusiastic fisherman, but after years of travelling along the rivers and lakes of our country he had collected and classified a fine collection of butterflies and beetles.

For Oleinikov nothing in life or in work or in public life was too trivial. A true and reliable friend, amusing and witty, he had a deep and subtle understanding of people, he was able quickly and accurately to distinguish between the apparent and the real, he was intolerant of any compromise that went against his conscience. It was natural that in a situation of moral uncertainty it was not easy for him, but nor was it always easy for people around him to deal with him. We must acknowledge that astonishing phenomenon of Vyacheslav Alekseevich Oleinikov: in critical situations by persistently and stubbornly championing his uncomfortable truth he very often won a victory, even though it was at the price of his peace of mind.

Moscow University has lost one of its dedicated colleagues, an outstanding teacher and gifted scientist, his wife and children have lost a true and loving husband and father, and all of us who knew him have lost a vital part of our everyday life. His shining memory lives on.

V.I. Arnol'd, E.A. Gorin, V.Ya. Lin,
Yu.V. Nesterenko, A.B. Shidlovskii,
M.A. Shubin, V.M. Tikhomirov,
P.L. Ul'yanov, A.G. Vitushkin

List of V.A. Oleinikov's published papers

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[2] The transcendence and algebraic independence of the values of certain E-functions, Vestnik Moskov. Univ. Ser. 1 **1962**, no. 6, 34–38. MR **26** # 1285.

[3] The transcendence and algebraic independence of values of E-functions representing a solution of a third-order linear differential equation, Dokl. Akad. Nauk SSSR **166** (1966), 540–543.
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[4] Algebraic independence of values of E-functions satisfying linear non-homogeneous differential equations of the third order, Dokl. Akad. Nauk SSSR **169** (1966), 32–34. MR **33** # 7303.
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[6] Algebraic functions with the simplest arrangement of singularities, Vestnik Moskov. Univ. Ser. 1 **1968**, no. 6, 9–13 (with A.N. Zhdanov). MR **40** # 7473.

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Translated by A. Lofthouse