1. In $\mathbb{Q}(\sqrt{6})$, we have the factorization

$$5 = (1 + \sqrt{6})(-1 + \sqrt{6}) = (71 + 29\sqrt{6})(-71 + 29\sqrt{6}).$$

Prove that $\pm 1 + \sqrt{6}$ and $\pm 71 + 29\sqrt{6}$ are irreducibles.
Verify that $(-71 + 29\sqrt{6})/(1 + \sqrt{6})$ is a unit.

2. Find $\gcd(-25 + 47i, 34 + 32i)$.

3. Factorize $7 + 4i$ in $\mathbb{Z}[i]$.

4. (a) Prove that $\gcd(y - 2i, y + 2i) = 1$ if $y$ is an odd integer.
   (b) Prove that $\gcd(y - i, y + i) = 1 + i$ if $y$ is an odd integer.
   (c) Show that the only solutions of the Diophantine equation

$$x^2 + 4 = y^3$$

are $x = \pm 11$, $y = 5$ and $x = \pm 2$, $y = 2$.

5. Show that the only solutions of the Diophantine equation

$$x^2 + 11 = y^3$$

are $x = \pm 4$, $y = 3$ and $x = \pm 58$, $y = 15$.

6. Show that the only solutions of the Diophantine equation

$$x^2 + 1 = 2y^3$$

are $x = \pm 1$, $y = 1$.

For Assignment 3, please hand in Question 4.