Problem Sheet 5, MP473, Semester 2, 2000

1. In $\mathbb{Q}(\sqrt{6})$, we have the factorization

$$5 = (1+\sqrt{6})(-1+\sqrt{6}) = (71+29\sqrt{6})(-71+29\sqrt{6}).$$

Prove that $\pm 1+\sqrt{6}$ and $\pm 71+29\sqrt{6}$ are irreducibles.
Verify that $(-71+29\sqrt{6})/(1+\sqrt{6})$ is a unit.

2. Find $\gcd(-25+47i, 34+32i)$.

3. Factorize $7+4i$ in $\mathbb{Z}[i]$.

4. (a) Prove that $\gcd(y-2i, y+2i) = 1$ if y is an odd integer.
   (b) Prove that $\gcd(y-i, y+i) = 1+i$ if y is an odd integer.
   (c) Show that the only solutions of the Diophantine equation

$$x^2+4 = y^3$$

are $x = \pm 11,\ y = 5$ and $x = \pm 2,\ y = 2$.

5. Show that the only solutions of the Diophantine equation

$$x^2+11 = y^3$$

are $x = \pm 4,\ y = 3$ and $x = \pm 58,\ y = 15$.

6. Show that the only solutions of the Diophantine equation

$$x^2+1 = 2y^3$$

are $x = \pm 1, y = 1$.

For Assignment 3, please hand in Question 4.