I am using the following lemma:

Lemma 1. *Suppose $u_j \leq v_j$ for $j = 1, \ldots, n$ and that for some J, we have $u_J < v_J$. Then*

$$\sum_{j=1}^{n} u_j < \sum_{j=1}^{n} v_j.$$

In our case, the a_{kj} are positive. So if we had $|x_J| < |x_k|$ for some J, then taking $u_j = a_{kj}|x_j|$ and $v_j = a_{kj}|x_k|$, we have $u_j \leq v_j$ for $j = 1, \ldots, n$ and

$$u_J = a_{kJ}|x_J| < v_J = a_{kJ}|x_k|,$$

and we would have, by the Lemma, a stronger inequality than (17), namely

$$|x_k| \leq \sum_{j=1}^{n} a_{kj}|x_j| < \sum_{j=1}^{n} a_{kj}|x_k| = |x_k|,$$

which gives the contradiction $|x_k| < |x_k|$.