ASSIGNMENT 3, MP204/MP274, 1st Semester 1999

Please hand in this assignment either in the tutorial *to the lecturer*, or *place in the box labelled* **MP204/274 ASSIGNMENT 3**, outside Room 424 Priestley Building, by 9am Monday 17th May 1999.

1. Let $A = \begin{bmatrix} -2 & -4 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & -4 & 0 & 0 \end{bmatrix}$.

   (a) Find $ch_A(x)$.

   (b) Prove that A is not diagonable.

2. Let $A = \begin{bmatrix} -1 & 1 & -1 \\ 0 & 1 & 0 \\ 2 & -1 & 2 \end{bmatrix} \in M_{3\times 3}(\mathbb{R})$.

   (a) Verify that $ch_A(x) = x(x-1)^2$.

   (b) Find $g_A(0)$ and $g_A(1)$ and prove that A is diagonizable over $\mathbb{R}$.

   (c) Find a non–singular matrix $P \in M_{3\times 3}(\mathbb{R})$ such that $P^{-1}AP = \operatorname{diag}(0, 1, 1)$.

3. Let $A \in M_{2\times 2}(\mathbb{R})$. Suppose $A^2 = 0, A \neq 0$.

   (a) Prove that $AX \neq 0$ for some $X \in \mathbb{R}^2$.

   (b) Let $AX_1 \neq 0$, where $X_1 \in \mathbb{R}^2$. Also Let $X_2 = AX_1$. Prove that X_1, X_2 are linearly independent.

   Moreover if $P = [X_1|X_2]$, prove that $P^{-1}AP = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$.

   (c) Determine $ch_A(x)$.

   (d) Explain why A is not diagonable over $\mathbb{R}$.

4. Let $A = \begin{bmatrix} 4 & -1 \\ 4 & 8 \end{bmatrix}$. Verify that $(A - 6I_2)^2 = 0$ and use the previous question to find a non–singular $P \in M_{2\times 2}(\mathbb{R})$ such that $P^{-1}AP = \begin{bmatrix} 6 & 0 \\ 1 & 6 \end{bmatrix}$.