ASSIGNMENT 2, MP204/MP274, 1st Semester 1999

Please hand in this assignment either in the tutorial *to the lecturer*, or *place in the box labelled* **MP204/274 ASSIGNMENT 2**, outside Room 424 Priestley Building, by 5pm Friday 16th April 1999.

1. Vectors $v_1 = (1,2,3)^t$, $v_2 = (2,5,3)^t$, $v_3 = (1,0,10)^t$ form a basis for $\mathbb{R}^3$. If $T : \mathbb{R}^3 \to \mathbb{R}^2$ is the linear transformation defined by

$$T(v_1) = (1,0)^t,\ T(v_2) = (1,0)^t,\ T(v_3) = (0,1)^t,$$

find $T(v)$, where $v = (1,1,1)^t$.

2. Let $V = M_{2\times 2}(\mathbb{R})$ and let $T : V \to \mathbb{R}$ be the mapping defined by

$$T(A) = a_{11} + a_{22},\ \text{where } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.$$

   (i) Prove that T is a linear transformation.

   (ii) Find a basis for $\operatorname{Ker} T$.

   (iii) Determine $\operatorname{rank} T$ and $\operatorname{nullity} T$.

3. Let $T : V \to V$ be a linear transformation. A *fixed point* of T is a vector v in V such that $T(v) = v$.

   (i) Prove that the set S of fixed points of T is a subspace of V.

   (ii) If $V = \mathbb{R}^2$ and $T = T_A$, where $A = \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{bmatrix}$, find a basis for S.

4. Let V be a vector space, $\dim V = 3$ and let $\beta : v_1, v_2, v_3$ be a basis of V. If $T : V \to V$ is the linear transformation determined by

$$T(v_1) = v_1 + v_2 + v_3,\ T(v_2) = v_1 - v_2 + v_3,\ T(v_3) = v_1 + 3v_2 + v_3,$$

find $[T]_\beta^\beta$ and hence, or otherwise, find bases for $\operatorname{Ker} T$ and $\operatorname{Im} T$. Also determine $\operatorname{rank} T$ and $\operatorname{nullity} T$.

5. (Compulsory question for MP274 students) Do one of (a) and (b):

   (a) $\mathcal{L}$ is the line $ax + by = 0$. $P = (x, y)$ is an arbitrary point of the (x, y) plane. If $Q = (x_1, y_1)$ is the foot of the perpendicular from P to $\mathcal{L}$, find a 2×2 matrix A such that

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}.$$

   (b) If $A \in M_{m\times n}(\mathbb{R})$, prove that $N(A^tA) = N(A)$ and deduce that $\operatorname{rank}(A^tA) = \operatorname{rank} A$. (Hint: If $Y = (y_1, \ldots, y_n)^t$, then $Y^tY = y_1^2 + \cdots + y_n^2$.)