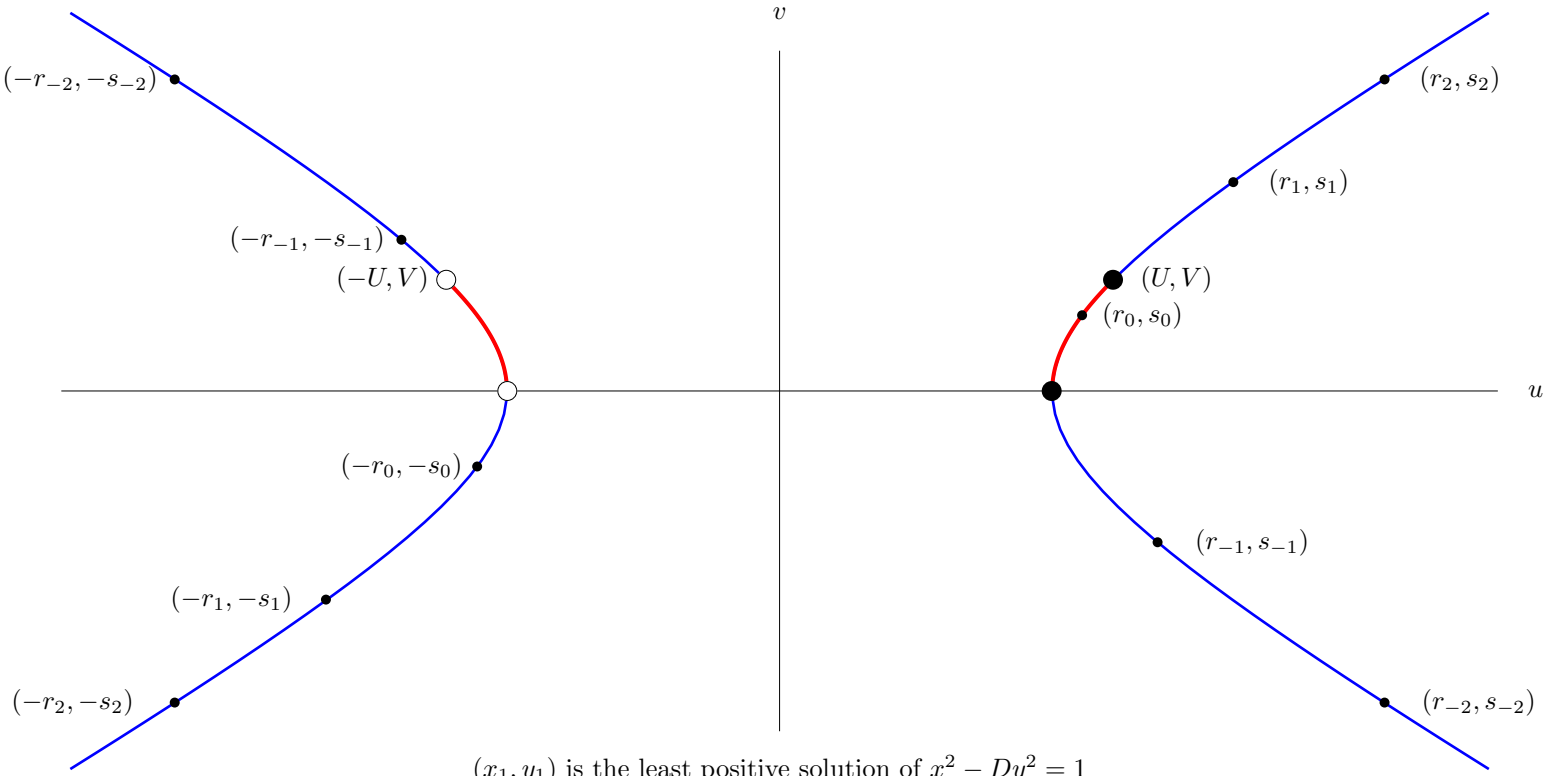


Equivalence class $\{(r_n, s_n)\}$, where $r_0^2 - Ds_0^2 = N > 0$



(x_1, y_1) is the least positive solution of $x^2 - Dy^2 = 1$

Nagell bounds: $U = \sqrt{\frac{N(x_1+1)}{2}}, V = y_1 \sqrt{\frac{N}{2(x_1+1)}}$

Fundamental solution region: $S = \{(u, v) \in \mathbb{Z} \times \mathbb{Z} | u^2 - Dv^2 = N, 0 < v < V\} \cup \{(\sqrt{N}, 0)\} \cup \{(U, V)\}$

$$r_n + s_n\sqrt{D} = (r_0 + s_0\sqrt{D})(x_1 + y_1\sqrt{D})^n, n \in \mathbb{Z}$$