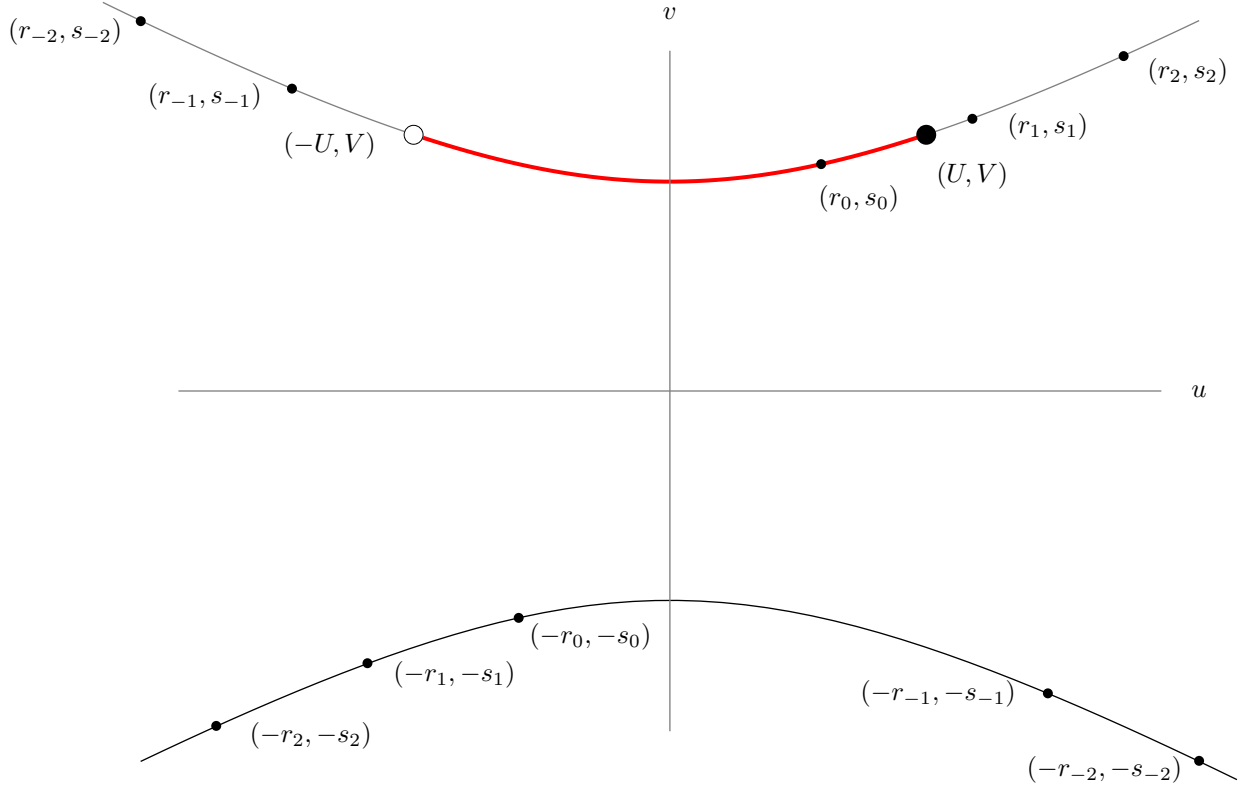


Equivalence class $\{(r_n, s_n)\}$, where $r_0^2 - Ds_0^2 = N < 0$



(x_1, y_1) is the least positive solution of $x^2 - Dy^2 = 1$

$$\text{Nagell bounds: } U = \sqrt{\frac{|N|(x_1-1)}{2}}, \quad V = y_1 \sqrt{\frac{|N|}{2(x_1-1)}}$$

Fundamental solutions region: $S = \{(u, v) \in \mathbb{Z} \times \mathbb{Z} \mid u^2 - Dv^2 = N, \sqrt{\frac{|N|}{D}} \leq v < V\} \cup \{(U, V)\}$

$$r_n + s_n\sqrt{D} = (u + v\sqrt{D})(x_1 + y_1\sqrt{D})^n, n \in \mathbb{Z}$$